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Research article

Accurate mathematical modeling of electronic load harmonics using machine learning software

Ahmad Al-Subhi

Electrical and Electronic Engineering Department, Yanbu Industrial College, Yanbu, Saudi Arabia

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This paper presents accurate mathematical models for several power system harmonics problems using an efficient machine learning tool, Eureqa. Three previously published research papers that included actual field measurements have been chosen to compare and assess the harmonics mathematical models presented in this paper. Following that, mathematical models of the output parameters as functions of the input parameters have been developed based on these data. For all these different research works and experiments, a total of 17 mathematical models have been built using basic curve fitting tools. Most of these proposed models couldn't fit the experimental data appropriately. Considerable error is observed for several models. In this paper, all these 17 problems are formulated using Eureqa software utilizing the same data presented in the discussed research works. Very accurate fitting capability to the experimental data is achieved using Eureqa, where almost near zero error is reached for the majority of the proposed models. The maximum mean absolute error (MAPE) among all developed models was 0.13% as opposed to 24% for the models presented in the literature.

1. INTRODUCTION

The fast emerging and growing of power electronics technology result in diverse types of new efficient and energy-saving home appliances. However, due to the electronics internal components of these appliances, external or internal switched-mode power supplies are necessary. Consequently, voltage and current waveforms are highly distorted due to the increasing use of such electronic appliances. Examples of such appliances include televisions (TVs), phone chargers, and personal computers (PCs). All these appliances need direct current power (DC) to operate. However, the commonly used power type available in the residential sector is AC. Hence, AC/DC converters are then essentially needed, which also have electronic components that lead to more distortions in the voltage waveform and, more significantly, the current waveform. On the other hand, the generated power from some renewable sources is DC such as photovoltaic (PV) and fuel cell (FC) systems which call for using some power electronics switches to be connected to the grid.[1–3] This supports the extensive need for some converters and inverters.

Generally, DC power systems have a wide area of study that needs some analysis and investigation. Efficiency, energy consumption, power quality, cost, and harmonics are some of the issues that have to be extensively analyzed and investigated. Specifically, harmonics issues caused by DC appliances have been presented in the literature to represent the impact of such appliances on some electrical parameters based on polynomials fitting to some experimental data.[4–7]

Significant and productive attempts have been presented to create models that, given an assumed operating point in real power consumption, estimate the input current harmonics of continuously variable electronic loads, such as variable speed drives (VSDs) and rectifier loads.[8–11] Specifically, the harmonic currents required by a variable speed drive or DC loads can be predicted given the known level of real power consumption and circuit parameters. An analytical solution to the inverse problem would be practical and even beneficial for many situations.[12,13] In other words, if it were possible to model and anticipate the actual power consumption of a drive from a line current harmonic pattern that was observed or approximated, it would allow for the development of new power monitoring and modeling strategies for limiting motor-drive design issues. Mathematical equations that, given observed or anticipated input harmonic levels, can precisely forecast real power consumption had been presented in [14]. The formulation was complicated and high harmonics have been ignored.

However, due to the lack of efficient and accurate curve fitting tools, the mathematical modeling approaches followed in such works are dependent on some basic curve fitting tools and some ready-built block polynomials. This results in inaccuracies in the obtained results, especially for highly nonlinear problems. Therefore, utilizing the most recent and efficient tools to improve the modeling capability is highly recommended. Following are the problems that are presented and discussed in this paper.

In [4], the impact of varying the number of connected PCs on the odd harmonics magnitudes has been formulated. The models are developed for the 3rd, 5th, 7th, and 9th harmonics as functions of the number of PCs. Reference

[5] investigates several factors affecting voltage and current Total Harmonics Distortions (THDs). These factors include experimental variations of supply voltage, source impedance magnitude, source impedance X/R ratio, system frequency, and background voltage distortion. Then, voltage and current THDs equations as functions of all these variables are formulated. In [6], power factor and voltage, and current THDs models have been built as functions of the number of connected electronic loads.

In this paper, all these problems are formulated using Eureqa software by utilizing the same experimental data used in these research works. Performance comparisons between the previously proposed models and Eureqa models had been discussed in terms of the models accuracy and fitting capability.

The paper is organized as follows. Section II discusses briefly the concept and modeling steps followed in Eureqa software. The experimental data overview and the developed models are presented in section II, while model comparisons and conclusions are shown in sections III and IV, respectively. It is important to state that the concentration in this paper is mainly focused on mathematical representations and modeling of harmonics, rather than deep power system harmonics analysis. The harmonics analysis concepts are extensively presented in [15–21].

2. EUREQA SOFTWARE: OVERVIEW

Eureqa is a mathematical tool that is created originally by Cornell's Creative Machines Lab and commercialized by Nutonian, Inc.[22,23] This software is used for the purpose of determining mathematical equations that describe sets of measured inputs and outputs in their simplest form. Detailed description and step by step user guide can be found in [24].

In Eureqa software, several simple and complex models are generated, in which the user has the option to choose the best model that fits the data with lowest error. The flow chart shown in Figure 1 illustrates the steps followed for mathematical formulation tasks using Eureqa. This software has been recently adopted in many applications.[25–27]

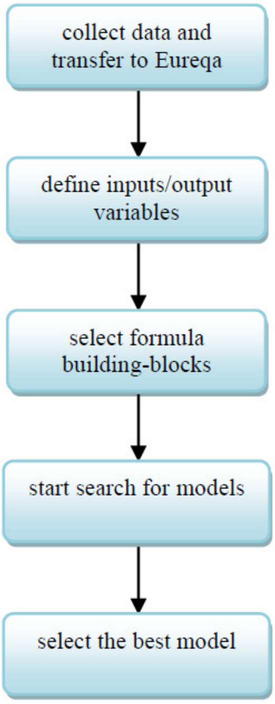

Fig. 1. Summary of steps followed in Eureqa to build mathematical models

3. MEASUREMENT DATA AND MATHEMATICAL MODELS

In this section, each problem is treated individually where the models proposed by each research work are listed. A brief experiment overview is discussed. Also, the models developed using Eureqa are presented in this section.

3.1. IMPACT OF THE NUMBER OF CONNECTED PCS ON THE ODD HARMONICS[4]

Table 1. Recorded results of PCs and harmonics magnitudes

No. of PCs	I_3 (%)	I_5 (%)	I_7 (%)	I_9 (%)
1	50	45	37	24
4	53	42	25	13
7	54	40	21	5
10	56	38	16	3
13	58	35	12	2
16	58	33	8	5
19	57	29	6	7
21	58	27	4	7
23	58	25	0	9

In the study presented in [4], the impact of successive PCs operation on the odd harmonics magnitudes of the supply current is presented. Mathematical relationships between the odd harmonics as output variables and the number of PCs as input variable have been formulated using some basic curve fitting tool utilizing fitting the data using linear and quadratic equations. 23 PCs are sequentially connected to the AC supply. Then, the current waveforms of odd harmonics have been captured and recorded in online measurements. Table 1 summarizes the results of these measurements where the first column lists the number of connected PCs and the subsequent columns demonstrate the individual percentage magnitudes of odd current harmonics corresponding to the number of connected PCs.

Therefore, using the data of Table 1, each output parameter has been formulated as a function of the number of PCs. As suggested by [4], the following equations describe the behavior of the odd harmonics based on the number of connected PCs variation:

$$I_3 = -0.0248N^2 + 0.935N + 49.228 \quad (1)$$

$$I_5 = -0.8961N + 46.239 \quad (2)$$

$$I_7 = -11.278\ln(N) + 39.85 \quad (3)$$

$$I_9 = 0.1188N^2 - 3.3403N + 25.159 \tag{4}$$

Where N is the number of connected PCs.

On the other hand, using Eureqa to model the same data given in Table 1, the following equations represent the best models obtained for each output parameter:

$$\begin{aligned} I_3 = {} & 48.2 + 1.09N + 0.966\sin(N) \\ & -0.117N\sin(N) + 0.125\sin(26.7N) \\ & -\frac{0.18}{\sin(26.7N)} + 0.0791N\sin(26.7N) \\ & -0.0293N^2 \end{aligned} \tag{5}$$

$$\begin{aligned} I_5 = {} & 44.9 + \frac{0.655}{\cos(N)} + 0.0884\cos(0.49N) \\ & +\frac{0.171\cos(N)}{N} + 0.000157N^2\cos(0.49N) \\ & -0.613N - 1.2\cos(N) - 0.0099N^2 \end{aligned} \tag{6}$$

$$\begin{aligned} I_7 = {} & 32.6 + \frac{3.56}{N} + \frac{0.00731}{\sin(N)} + 0.000843N^3 \\ & +3.97\cos(-0.661N) \\ & +0.0277N^2\cos(-0.661N) \\ & -1.76N - 0.689N\cos(-0.661N) \end{aligned} \tag{7}$$

$$\begin{aligned} I_9 = {} & 29.7 + 0.374N^2 + 0.145\cos(26.4N) \\ & -5.66N - 0.414\sin(N) - 0.00724N^3 \\ & -0.0357N\sin(N) - 0.834\cos(26.4N)^2 \end{aligned} \tag{8}$$

For all Eureqa equations shown in this paper, the coefficients are rounded to 3 to 5 decimals due to the limited space. However, the simulations are performed based on the entire numbers as obtained by Eureqa.

3.2. IMPACT OF SEVERAL PARAMETERS VARIATIONS ON VOLTAGE AND CURRENT THD'S[5]

The study presented in [5] discusses the impact of changing five parameters on voltage and current THDs. These parameters are the supply voltage, source impedance magnitude, source impedance *X/R* ratio, system frequency, and background voltage distortion. Voltage and current THDs mathematical equations are formulated as functions of each of these parameters using a basic curve fitting tool. The 10 models proposed by [5] are as follows:

$$THD_V = -0.0059V_S + 2.6846 \tag{9}$$

$$THD_I = 0.2242V_S + 35.794 \tag{10}$$

$$\begin{aligned} THD_V = {} & 6.322Z_s^4 - 14.529Z_s^3 + 11.059Z_s^2 \\ & +0.8777Z_s + 0.5808 \end{aligned} \tag{11}$$

$$THD_I = 5.319Z_s^2 - 23.962Z_s + 95.363 \tag{12}$$

$$\begin{aligned} THD_V = {} & 0.0000007K^3 + 0.0008K^2 \\ & -0.0042K + 1.2345 \end{aligned} \tag{13}$$

$$THD_I = -0.001K + 89.384 \tag{14}$$

$$THD_V = 0.0096f + 0.7898 \tag{15}$$

$$THD_I = -0.224f + 100.9 \tag{16}$$

$$\begin{aligned} THD_V = {} & 0.0032V_D^4 - 0.0453V_D^3 + 0.2275V_D^2 \\ & +0.4314V_D + 1.2672 \end{aligned} \tag{17}$$

$$\begin{aligned} THD_I = {} & -0.0588V_D^3 + 0.827V_D^2 \\ & -7.9271V + 89.773 \end{aligned} \tag{18}$$

Where V_S is the supply voltage, Z_S is the source impedance magnitude, K is source impedance *X/R* ratio, f is the system frequency and V_D is the background voltage distortion.

Moreover, the models proposed by Eureqa are represented by the following equations:

$$\begin{aligned} THD_V = {} & \frac{0.00514}{\sin(0.197V_S)} \\ & +\frac{230774.35 + 14\sin(0.197V_S)}{V_S^2} \\ & -8.24 - 0.0744\sin(0.197V_S) \\ & +0.00228V_S \end{aligned} \tag{19}$$

$$\begin{aligned} THD_I = {} & 30.6 + 0.248V_S + 0.495\sin(V_S) \\ & -\frac{0.704}{\sin(V_S)} + 0.174\cos\left(0.00111V_S^2\right) \end{aligned} \tag{20}$$

$$\begin{aligned} THD_V = {} & 0.0328 + 4.66Z_S \\ & +0.0477\sin(81.6Z_S) \\ & +\frac{0.0258 - 0.0258\sin(81.7Z_S)}{Z_S} \\ & -0.421Z_s^3 \end{aligned} \tag{21}$$

$$\begin{aligned} THD_I = {} & 81.6 + 3.24Z_S + \frac{2.87}{Z_S} \\ & +3.8Z_S^5 - \frac{0.207}{Z_S^2} - 14.3Z_s^2 \end{aligned} \tag{22}$$

$$\begin{aligned} THD_V = {} & 0.76 + 0.00163\sin(K) \\ & -0.0332\sin(K)^2 + \frac{1.87\sin(K)^2}{K} \\ & -0.0000015K^3 + 0.031K \end{aligned} \tag{23}$$

$$\begin{aligned} THD_I = {} & 91 - \frac{6.32}{K} + 0.118\sin(K) \\ & +0.000121K^2 + 0.00000612K^3 \\ & -0.0588K \end{aligned} \tag{24}$$

$$\begin{aligned} THD_V = {} & 0.00196\cos(f) + 0.000756\cos(f)^2 \\ & -0.00229f - 0.00579\sin(14.1f) + 1.38 \end{aligned} \tag{25}$$

$$\begin{aligned} THD_I = {} & 81.9 + 0.367f + 0.244\cos(f)^2 \\ & +0.315\sin(2.79 - 44.9f) - 22.8\cos(f) \end{aligned} \tag{26}$$

$$\begin{aligned} THD_V = {} & 1.27 + 0.8V_D + 0.00028V_D^2 \\ & +0.0248\sin\left(V_D^2\right) - 0.292\sin\left(V_D^2\right)^2 \end{aligned} \tag{27}$$

$$\begin{aligned} THD_I = {} & 88.5 + 0.311V_D^2 + 1.44\cos(1.57V_D) \\ & -6.42V_D + 0.234V_D^2\cos(1.57V_D) \\ & -1.25V_D\cos(1.57V_D) \end{aligned} \tag{28}$$

3.3. IMPACT OF THE NUMBER OF CONNECTED PCS ON VOLTAGE AND CURRENT THD'S AND POWER FACTOR[6]

In the study presented in [6], 23 PCs are operated one by one where the measured parameters include power factor and voltage and current THDs. The modeling concentrates on building mathematical relationships that relate these parameters as functions of the number of PCs. The data for all 23 PCs are available in this study. Table 2 illustrates the data obtained from the measurements. Since the measured data are almost linear, all models proposed by [6] are based on fitting linear functions to the data. Two-point form of the equation of the line is used for all parameters as follows[6]:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \tag{29}$$

Where *x* represents any input parameter which is the number of connected PC's in this work and *y* represents any output parameter which is represented by the power factor and the total harmonic distortions of voltage and current.

The mathematical models proposed by [6] are represented by the following equations:

$$THD_I = -0.81N + 80.11 \tag{30}$$

$$THD_V = 0.1818N + 2.518 \tag{31}$$

Table 2. Recorded results of PCs and harmonics magnitudes along with power factor and voltage and current THDs[6]

No. of PCs	THD_I (%)	THD_V (%)	*power factor*
1	79.3	2.7	0.6
2	78.2	2.9	0.61
3	76	3	0.63
4	74.6	3.1	0.64
5	74.2	3.2	0.65
6	73.7	3.3	0.66
7	72.3	3.4	0.67
8	71.5	3.5	0.68
9	70.7	3.6	0.68
10	70.1	3.7	0.69
11	69.7	3.9	0.69
12	69.2	4.1	0.7
13	68.6	4.4	0.7
14	67.9	4.7	0.71
15	67.1	4.9	0.71
16	66.2	5.2	0.72
17	65.4	5.5	0.72
18	64.7	5.7	0.73
19	64	5.9	0.73
20	63.5	6.1	0.74
21	62.8	6.3	0.74
22	62.1	6.5	0.75
23	61.4	6.7	0.75

$$PF = 0.006818N + 0.5931 \tag{32}$$

In addition, the models formulated using Eureqa take the following forms of equations:

$$\begin{aligned} THD_I = {} & 76.9 + 0.0748\cos(N) + \frac{40.4}{N^3} \\ & -0.252\cos(0.693N) - \frac{37.1}{N^4} \\ & + \frac{0.00000254N^2}{\cos(N)} - 0.65N \\ & -0.00000254N^4 \end{aligned} \tag{33}$$

$$\begin{aligned} THD_V = {} & 0.109N + 0.000163N \\ & +0.369\sin(0.297N) \\ & -0.413N\sin(0.000464 + 0.297N) \\ & +2.49 - \sin(0.04\cos(\sin(0.297N))) \\ & * \cos\left[\frac{2760}{\sin(0.000464 + 0.297N)}\right] \end{aligned} \tag{34}$$

$$\begin{aligned} PF = {} & 0.705 + 0.00081N - \frac{0.388}{N} + \frac{0.518}{N^2} \\ & - \frac{0.238}{N^3} + 0.0000865N^2 \\ & -0.003\sin(2.3\sin(3.01N + \cos(\sin(N)))) \end{aligned} \tag{35}$$

Where *PF* is the power factor, THD_V and THD_I are the THD for voltage and current, respectively.

4. MATHEMATICAL MODELS ANALYSIS AND COMPARISON

Analysis and comparisons of all models are presented in this section. These include individual error calculations, performance comparisons, and mean absolute error (*MAPE*) computations. MAPE is evaluated using the following equation:

$$\mathrm{MAPE} = 100 \times \frac{1}{n}\sum_{k=1}^{n}\frac{|Xa(k) - Xm(k)|}{Xa(k)} \tag{36}$$

Where X_a and X_m denote the actual and modeled parameters, respectively. X represents any of the modeled parameters and n is the number of points. All models are evaluated and compared with the measured data in order to check the fitting capability of the developed models.

4.1. EUREQA MODELS COMPARISON WITH MODELS PROPOSED BY [4]

All models are evaluated concerning the number of connected PCs to assess the ability of the proposed models to fit the experimental data. Figure 2 shows the actual measured output parameters and the modeled parameters using Eureqa and using the curve fitting tool presented in [4].

Figure 2 demonstrates that the built models using Eureqa have successfully and accurately followed the actual behavior of all parameters when subjected to changes by varying the number of connected PCs. The models proposed by [4] are successful in some points while they are not fitted correctly for many other points. To precisely assess the proposed models, Figure 2 also shows the absolute percentage of error which is calculated at each point according to the results obtained from actual measurements.

Using the models proposed in [4], low and acceptable error values are obtained for the case of 3rd and 5th harmonics modeling. However, considerable errors are noticed at many points of the 7th and 9th harmonics. On the other hand, the models developed by Eureqa have shown almost zero or negligible error values. The maximum evaluated error over all parameters is less than 0.02%. Therefore, these results conclude the accuracy and superiority of Eureqa software to build accurate models relating input and output variables in a fashion very identical to the results obtained from measurements.

Table 3 lists the *MAPE* values for all models as compared with the measured data. The table clearly shows the superiority of the models developed by Eureqa over the models proposed by [4].

4.2. EUREQA MODELS COMPARISON WITH MODELS PROPOSED BY [5]

The models proposed by [5] as well as models developed by Eureqa are evaluated at their respective input variables. Figures 3 to 7 show the models comparison for all parameters analyzed in [5] along with models developed by Eureqa.

Although most of the models proposed by [5] show efficient performance fitting capability with low error, Eureqa models have shown better performance with almost zero

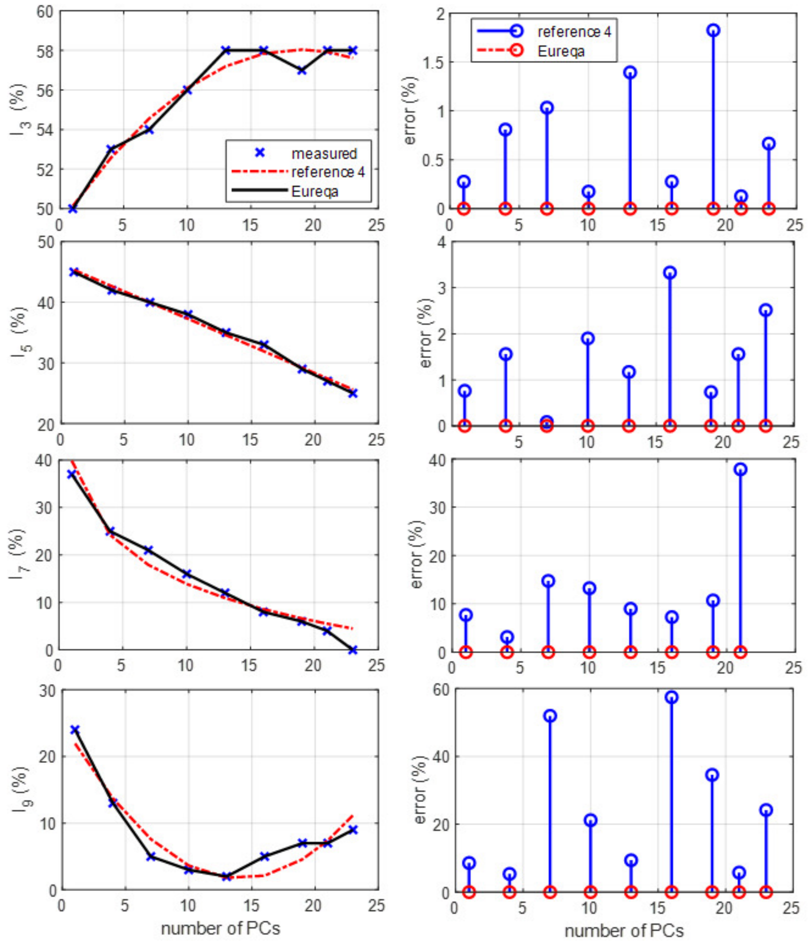

Fig. 2. Eureqa models along with reference [4] models comparison

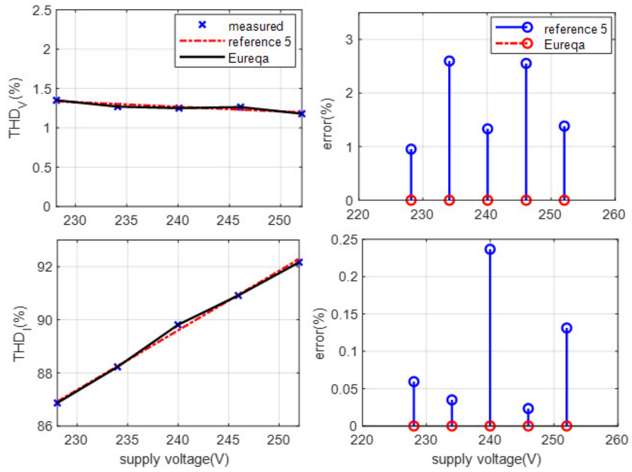

Fig. 3. Eureqa models along with reference [5] models comparison, THD's vs. supply voltage

error. Table 4 lists the *MAPE* values for all models developed to formulate the output parameters.

Table 3. MAPE comparisons for Eureqa and reference [4] models

	MAPE (%)	
parameters	reference [4]	Eureqa
I_3	0.731347	0.000122
I_5	1.512746	0.000048
I_7	12.95257	0.001754
I_9	24.27790	0.001325

4.3. EUREQA MODELS COMPARISON WITH MODELS PROPOSED BY [6]

In the case of comparison with the models presented in [6], all proposed models are evaluated at the number of PCs from 1 to 23 as input variables. Figure 8 depicts the comparison between the output parameters profiles, in addition to the absolute percentage error profiles for all models when compared with the actual data.

Figure 8 shows that the models proposed using Eureqa are more accurate and superior to all models proposed using the approach followed in reference [6].

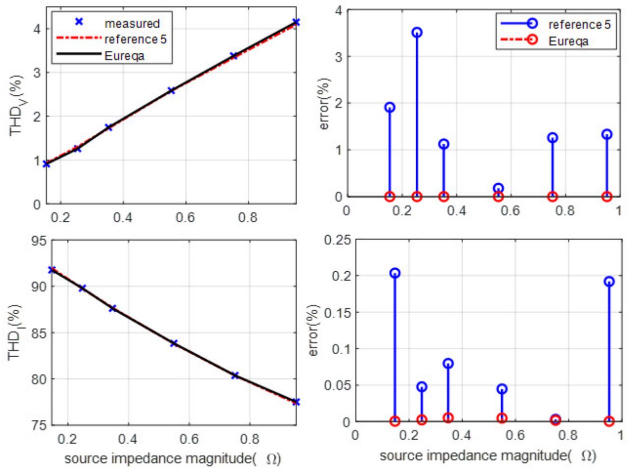

Fig. 4. Eureqa models along with reference ^{5} models comparison, THD's vs. source impedance magnitude

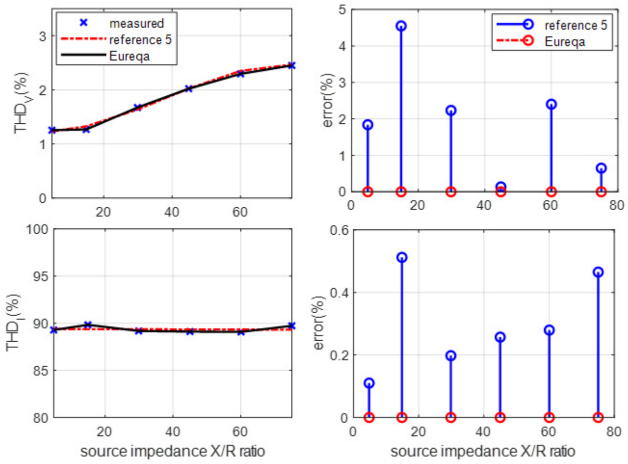

Fig. 5. Eureqa models along with reference ^{5} models comparison, THD's vs. source impedance *X/R* ratio

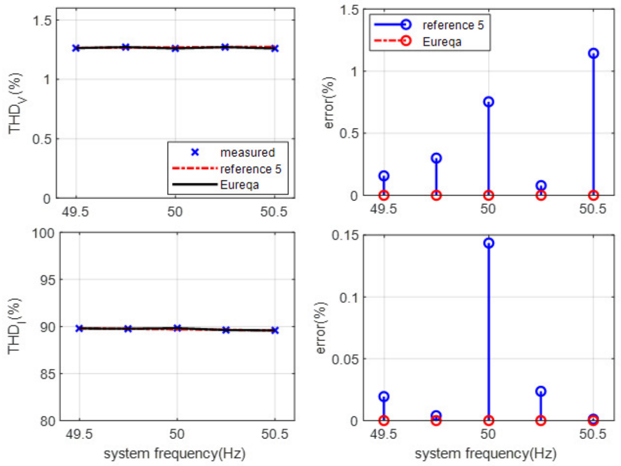

Fig. 6. Eureqa models along with reference ^{5} models comparison, THD's vs. system frequency

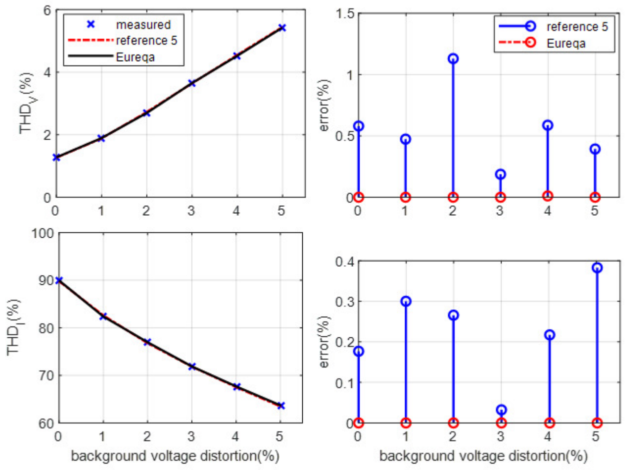

Fig. 7. Eureqa models along with reference ^{5} models comparison, THD's vs. back distortion voltage

Table 5 compares the *MAPE* values of all output parameters models. The table clearly shows the superiority of the models proposed by Eureqa over the models proposed by ^{6}.

5. CONCLUSION

In this paper, accurate mathematical models using Eureqa software have been presented for different selected power system harmonics problems. The software shows efficient, superior, and more accurate performance than all models proposed by different research works. Therefore, the modeling capability for the discussed problems has been improved significantly using Eureqa as compared with the basic curve fitting tools employed in the presented research works. Where its applications are not limited for only harmonics modeling, this machine learning tool is easy and friendly used and may also be adapted for different electrical power system problems as well as other problems including chemical, biological, and forecasting problems. There is a significant and obvious contrast between the MAPE values for the proposed models and what have been done in the literature, with an error rate that does not surpass 0.13% as opposed to the literature, which reached 24%.

Table 4. MAPE comparisons for Eureqa and reference [5] models

		MAPE (%)	
input parameters	output parameters	reference [5]	Eureqa
V_S	THD_V	1.7638801	0.00001126
V_S	THD_I	0.0970823	0.00000202
Z_S	THD_V	1.5534893	0.00003774
Z_S	THD_I	0.0951393	0.00257605
K	THD_V	1.9680202	0.00000469
K	THD_I	0.3034505	0.00000252
f	THD_V	0.4863297	0.00000109
f	THD_I	0.0383584	0.00000071
V_D	THD_V	0.5587784	0.00181279
V_D	THD_I	0.2291687	0.00000932

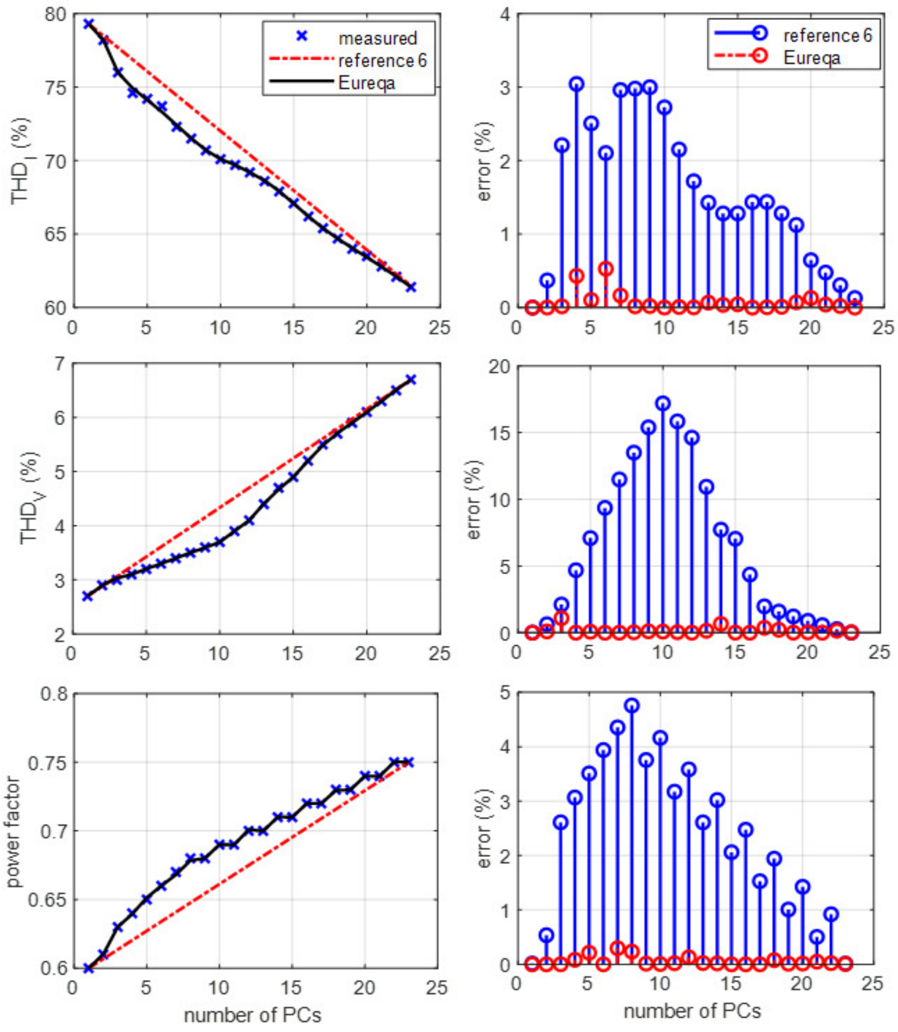

Fig. 8. Eureqa models along with reference [6] models comparison

Table 5. MAPE comparisons for Eureqa and reference [6] models

	MAPE (%)	
parameters	reference [6]	Eureqa
THD_I	1.591278	0.075010
THD_V	6.457097	0.136866
PF	2.389672	0.052426

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